

Pentaquarks: Status and Perspectives for Lattice Calculations

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The present status of pentaquark spectroscopy in lattice QCD is reviewed. This talk also includes a brief introduction of pentaquark baryons.

1. Introduction

The quantum chromo-dynamics (QCD) may not preclude the presence of the multi-quark hadrons such as tetraquark ($qq\bar{q}\bar{q}$), pentaquark ($qqqq\bar{q}$), dibaryon ($qqqqqq$) and so on, because of the color confinement. Especially, we are let to be interested in *exotic multi-quark hadrons*, which should have *exotic* quantum numbers. Now, let us address ourself to the pentaquark state. Consider the $SU(3)$ flavor case. Pentaquark states should form six different multiplets:

$$\begin{aligned} 3_f \otimes 3_f \otimes 3_f \otimes 3_f \otimes \bar{3}_f \\ = 1_3 \oplus 8_8 \oplus 10_4 \oplus \bar{10}_2 \oplus 27_3 \oplus 35_1 \end{aligned}$$

where subscripts in the right hand side denote the number of degeneracy in each multiplet. The first three multiplets are common in the case of usual baryons. However, the last three multiplets; antidecuplet, 27-plet and 35-plet, are distinct irreducible representations since those multiplets have an apparent exotic quantum-number such as *strangeness* $+1$. Needless to say, $S = +1$ baryon can not be accommodated by usual baryons. Other possible exotic quantum-numbers are represented as stars in Fig. 1. Of course, one cannot predict which multiplet is preferred for the possible $S = +1$ pentaquark baryon, within the group theoretical argument. The Skyrme model [1] and the chiral soliton model [2], however, predict that the lowest $S = +1$ state appears uniquely in the antidecuplet and its spin and parity is spin-half and positive parity.

Recently, the LEPS collaboration at Spring-8 has observed a very sharp peak resonance in the K^- missing-mass spectrum of the $\gamma n \rightarrow nK^+K^-$

reaction on ^{12}C [3]. The observed resonance should have strangeness $+1$. Thus, $\Theta^+(1540)$ cannot be a three quark state and should be an exotic baryon state with the minimal quark content $uudd\bar{s}$. The peak position is located at 1540 MeV with a very narrow width. Those are quite consistent with the chiral-soliton model's prediction [2]. This discovery is subsequently confirmed by other experiments [4,5]¹. Experimentally, spin, parity and isospin are not determined yet. Non-existence of a narrow resonance in pK^+ channel indicates that possibility of $I = 1$ has been already ruled out [3,5]

Many theoretical studies of pentaquarks are also triggered by the discovery of the $\Theta^+(1540)$. I introduce the most reputed model proposed by Jaffe and Wilczek [7]. In the naive quark models, the low-lying pentaquark state should have spin-1/2 and negative parity. However, in this case, the pentaquark baryon just falls apart into KN in a S-wave. It is difficult to explain its very narrow width. Jaffe and Wilczek propose a simple idea to flip the parity of the low-lying pentaquark. Suppose there is the strong diquark correlation. The spin-0, color triplet and flavor triplet diquark would be favored within the simple one gluon exchange. The pentaquark can be composed of two identical bosons (diquarks) and one antiquark. However, the anti-symmetrization in terms of color, requires relative odd number's angular-momentum between the pairs of identical bosons. Otherwise, the wave function of the pentaquark state should be vanished. Resulting parity of the low-lying pentaquark is same as the

¹It should be noted, however, that the experimental evidence for the $\Theta^+(1540)$ is not very solid yet since there are a similar number of negative results to be reported [6].

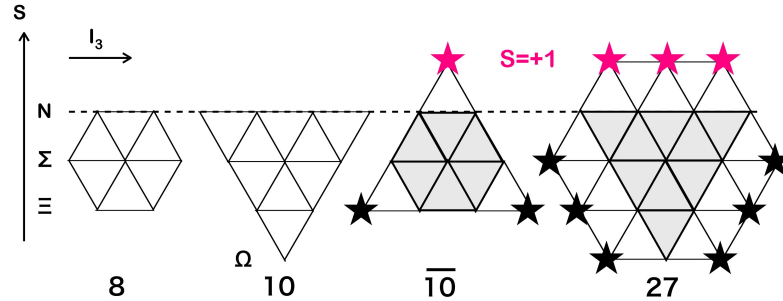


Figure 1. Weight diagrams for possible pentaquark multiplets. Stars represent the states which have exotic quantum numbers (electric charge and strangeness).

chiral soliton model [7]. In addition, the $S = +1$ baryon is uniquely assigned to antidecuplet in this description.

Consequently, correlated quark models, *i.e.* the diquark model, may accommodate the positive-parity isosinglet pentaquark $\Theta(uudd\bar{s})$ as same as the chiral soliton model. However, there are essential differences between the diquark model and the chiral soliton model. In correlated quark models, one cannot fully discriminate the other multiplet such as octet. If apart from the $SU(3)$ flavor limit, the mixing between octet and antidecuplet should be taken into account. Jaffe and Wilczek advocate the ideal mixing case, which is favored in the vector meson spectrum [7]. The ideal mixing provides the different prediction for the mass of possible exotic $\Xi_{3/2}^-$ state, which should have isospin-3/2. Furthermore, the Roper resonance, $N(1440)$, can be accommodated in the diquark model owing to the ideal mixing [7].

A candidate of the isospin-3/2 pentaquark state has been reported by the NA49 collaboration [8]. They observed the sharp peak resonance, which has exotic quantum numbers, in $\Xi\pi$ missing mass analysis. The observed mass is located somewhat between predictions made by the chiral soliton model and the diquark model. In this experiment, the isospin partner, $\Xi_{3/2}^0$ is also observed. It should be pointed out that this discovery is not confirmed yet by other experiments [6].

I shortly mention about charm or bottom analog of the Θ state. If anti-strange quark is re-

placed by a heavier antiquark, what is going to be happened. Several models predict that charm or bottom analog of the pentaquark is expected to be a bound state [7,9]. Recently, the discovery of the exotic anti-charmed baryon ($uudd\bar{c}$) has been announced by the H1 collaboration [10]. However, the observed state is higher than the DN threshold and also the D^*N threshold against several model predictions [7,9]. Again, this discovery has not been confirmed yet by other experiments.

2. Lattice pentaquark spectroscopy

If the pentaquark baryons really exist, such states must emerge directly from first principles, QCD. Of course, what we should do is to confirm the presence of the pentaquarks by lattice QCD. Experimentally, it is rather difficult to determine the parity of the $\Theta^+(1540)$. Thus, lattice QCD has a chance to answer the undetermined quantum numbers before experimental efforts. Lattice QCD has also a feasibility to predict the masses for undiscovered pentaquark baryons. I stress that there is substantial progress in lattice study of excited baryons recently [11]. Especially, the negative parity nucleon $N^*(1535)$, which lies close to the $\Theta^+(1540)$, has become an established state in quenched lattice QCD [11,12]. Here I report that quenched lattice QCD is capable of studying the $\Theta^+(1540)$ as well.

Indeed, it is not so easy to deal with the $qqqq\bar{q}$ state rather than usual baryons (qqq) and

mesons ($q\bar{q}$) in lattice QCD. The $qqqq\bar{q}$ state can be decomposed into a pair of color singlet states as qqq and $q\bar{q}$, in other words, can decay into two hadron states even in the quenched approximation. For instance, one can start a study with a simple minded local operator for the $\Theta^+(1540)$, which is constructed from the product of a neutron operator and a K^+ operator such as $\mathcal{O} = \varepsilon_{abc}(d_a^T C \gamma_5 u_b) d_c (\bar{s}_e \gamma_5 u_e)$. The two-point correlation function composed of this operator, in general, couples not only to the Θ state (single hadron) but also to the two hadron states such as an interacting KN system [13,14]. Even worse, when the mass of the $qqqq\bar{q}$ state is higher than the threshold of the hadronic two-body system, the two-point function should be dominated by the two hadron states. Thus, a specific operator with as little overlap with the hadronic two-body states as possible is desired in order to identify the signal of the pentaquark state in lattice QCD. Once one can identify the pentaquark signal in lattice QCD, to determine the parity of the $\Theta^+(1540)$ is the most challenging issue at present. Thus, it is necessary to project out the parity eigenstate from given lattice data precisely. In the following subsections, I discuss three related issues

2.1. Estimation of the KN threshold

The experimentally observed $\Theta^+(1540)$ state is clearly a resonance state. However, its mass is near the KN threshold. We could manage to calculate the pentaquark as a bound state *if its parity were positive*.

I recall that all momenta are quantized as $\vec{p}L = 2\pi\vec{l}$ ($\vec{l} \in \mathbf{Z}^3$) on lattice in finite volume (the spacial extent L) with the periodic boundary condition (PBC).² Thus, the spectrum of energies of two hadron states such as KN states with zero total momentum should be discrete and these energies are approximately equal to values, which are evaluated in the noninteracting case:

$$E_n^{KN} = \sqrt{M_N^2 + p_n^2} + \sqrt{M_K^2 + p_n^2} \quad (1)$$

²Of course, this quantization condition may change to $\vec{p}L + 2\delta(p) = 2\pi\vec{l}$ with a scattering phase shift $\delta(p)$ by an interaction between two hadrons. However, the KN channel has very weak interactions as is well known experimentally. One may omit this shift for a crude estimation.

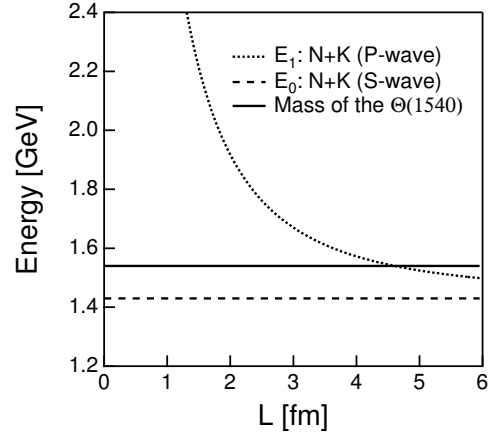


Figure 2. The S-wave and P-wave KN threshold energies on a lattice of spatial extent L . In the case of $L \leq 4.6$ fm, the mass of the $\Theta^+(1540)$ is lower than the P-wave KN threshold.

where $p_n = \sqrt{n} \cdot 2\pi/L$ and $n \in \mathbf{Z}$. The positive parity Θ state decay into KN in a P-wave where the KN system should have a nonzero relative momentum. The P-wave KN threshold is simply estimated at an energy level E_1 , which is evaluated with the smallest nonzero momentum $p_1 = 2\pi/L$ in Eq. (1). This energy level E_1 can be lifted by decreasing spatial extent as depicted in Fig. 2 while the lowest energy level E_0 , which corresponds to the S-wave KN threshold, remains unchanged. The level crossing between E_1 and the Θ mass takes place around 4.6 fm in this crude estimation. It implies that *the positive parity Θ state may become a bound state in the typical lattice size of currently available lattice calculations, i.e. $L \approx 2 - 3$ fm.*

2.2. Choice of operators

For the case of the negative parity Θ state, the presence of the KN scattering state must complicate the study of pentaquarks in lattice QCD. One should choose an optimal operator, which couples weakly to the KN scattering state, in order to access the pentaquark state above the (S-wave) KN threshold.

For this direction, I would like to recall that

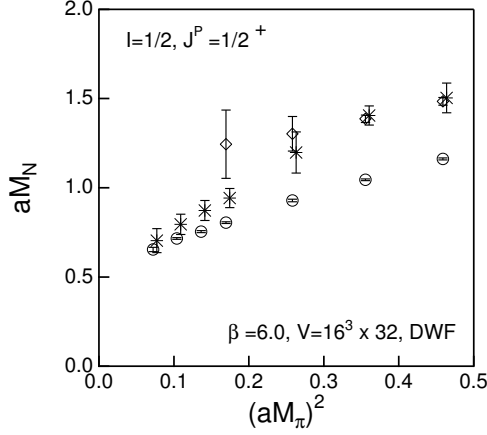


Figure 3. Comparison of the fitted mass from $\langle \mathcal{O}_1^N \bar{\mathcal{O}}_1^N \rangle$ (\circ), $\langle \mathcal{O}_2^N \bar{\mathcal{O}}_2^N \rangle$ (\diamond) and the cross correlation $\langle \mathcal{O}_1^N \bar{\mathcal{O}}_2^N + \mathcal{O}_2^N \bar{\mathcal{O}}_1^N \rangle$ ($*$) [12].

the less known observation in the spectroscopy of the nucleon. There are two possible interpolating operators for the $I = 1/2$ and $J^P = 1/2^+$ state; $\mathcal{O}_1^N = \varepsilon_{abc}[u_a^T C \gamma_5 d_b] u_c$ and $\mathcal{O}_2^N = \varepsilon_{abc}[u_a^T C d_b] \gamma_5 u_c$, even if one restricts operators to contain no derivatives and to belong to the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ chiral multiplet under $SU(2)_L \otimes SU(2)_R$ [12]. Of course, two operators have the same quantum number of the nucleon. The first operator \mathcal{O}_1^N is utilized conventionally in lattice QCD since the second operator \mathcal{O}_2^N vanishes in the non-relativistic limit. It implies that the second operator is expected to have small overlap with the nucleon $|\langle 0 | \mathcal{O}_2^N | \text{Nucl} \rangle| \approx 0$. Indeed, the mass extracted from the correlator constructed by the second operator \mathcal{O}_2^N exhibit the different mass from the nucleon mass [12,15] as shown in Fig. 3. The operator dependence on a overlap with desired state is evident, at least, in the heavy quark regime, while the cross correlation suggests that the small overlap with the nucleon might be no longer robust in the light quark regime where is far from the non-relativistic description [12].

2.3. Parity projection

The intrinsic parity of the *local* baryon operator can be defined by the parity transformation of

internal quark fields as

$$\mathcal{P} \mathcal{O}^{(\eta)}(\vec{x}, t) \mathcal{P}^\dagger = \eta \gamma_4 \mathcal{O}^{(\eta)}(-\vec{x}, t), \quad (2)$$

where $\eta = \pm 1$. However, due to the relation $\mathcal{O}^{(+)} = \gamma_5 \mathcal{O}^{(-)}$ for the *local* baryon operator, the resulting two-point correlation functions are also related with each other as $\langle \mathcal{O}^{(+)}(x) \bar{\mathcal{O}}^{(+)}(0) \rangle = -\gamma_5 \langle \mathcal{O}^{(-)}(x) \bar{\mathcal{O}}^{(-)}(0) \rangle \gamma_5$. This means that the two-point correlation function composed of the *local* baryon operator can couple to both positive- and negative-parity states. However, I note that *anti-particle contributions of opposite parity states is propagating forward in time*. Thus, the $+/-$ parity eigenstate in the forward propagating contributions is obtained by choosing the appropriate parity projection $(1 \pm \eta \gamma_4)/2$, which is given in reference to the intrinsic parity of operators, η . Details of the parity projection are described in Ref. [12].

3. First exploratory studies

3.1. Local pentaquark operators

As I remarked previously, an optimal operator, which couples weakly to the KN scattering state, would be required to explore the pentaquark baryons in lattice QCD. For this purpose, two types of local pentaquark operator for isosinglet state are proposed in the first two studies. One is a *color variant* of the simple product of nucleon and kaon operators,

$$\mathcal{O}_{I=0}^{(-)} = \varepsilon_{abc}[u_a^T C \gamma_5 d_b] \{u_e(\bar{s}_e \gamma_5 d_c) - (u \leftrightarrow d)\}, \quad (3)$$

which is proposed by Csikor *et al.* [16]

The other is proposed by Sasaki [17] as in a rather exotic description guided by the diquark-diquark-antiquark structure:

$$\mathcal{O}_{I=0}^{(\eta)} = \varepsilon_{abc} \varepsilon_{aef} \varepsilon_{bgh} [u_e^T C \Gamma_1 d_f] [u_g^T C \Gamma_2 d_h] C \bar{s}_c^T \quad (4)$$

where $\Gamma_{1,2} = 1, \gamma_5, \gamma_5 \gamma_\mu$ (but $\Gamma_1 \neq \Gamma_2$) and the superscript “ η ” stands for the intrinsic parity of the operator. There are three kinds of diquark-diquark-antiquark operator in this description, which are useful for the extended study with the 3×3 correlation matrix analysis. More details of construction of the diquark-diquark-antiquark operator are described in Ref. [17].

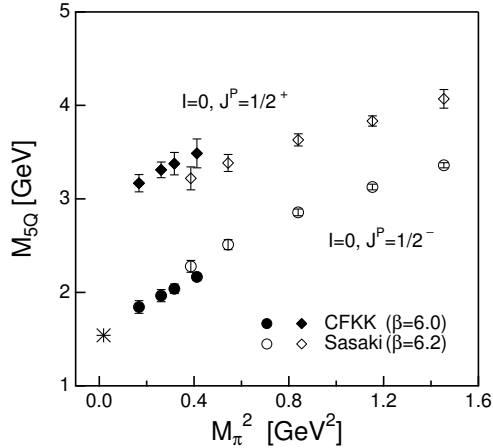


Figure 4. Masses of the isosinglet $S = +1$ baryons with both positive- and negative-parity as functions of pion mass squared [16,17]. The experimental value for $\Theta^+(1540)$ is marked with a star.

In an exploratory study, one may assume that those interpolating operators have smaller overlap with the KN -scattering state than the simple product of nucleon and kaon operators, at least, in the heavy quark regime. Because, in the non-relativistic limit, all of them give rise to the different wave function from the KN two-hadron system.

3.2. Results

The first two lattice studies were performed with the Wilson gauge action and the Wilson fermion action at the almost same box size $L \simeq 2.0 - 2.2$ fm. The lattice spacing Csikor *et al.* use is rather coarse than that of Sasaki, but their calculation was employed with relatively lighter pion masses ($M_{\pi} \sim 0.4 - 0.6$ GeV). The main difference between two studies is the choice of pentaquark operators.

After some initial confusion about the parity assignment³, both calculations agreed that the lowest state of the isosinglet $S = +1$ baryons has the negative parity as shown in Fig. 4. The main results from the first two exploratory studies can

be summarized as follows.

- The current lattice simulations seem to give no indication of a pentaquark in the positive parity channel to be identified with the $\Theta^+(1540)$.
- The negative parity channel can easily accommodate a pentaquark with a mass close to the experimental value.

Therefore, both authors conclude that the exploratory lattice study favors spin-parity $(1/2)^-$ and isospin 0 for the $\Theta^+(1540)$. In Ref. [17], the anti-charmed analog of the Θ state was also explored. It is found that the $\Theta_c(uudd\bar{c})$ lies much higher than the DN threshold, in contrast to several model predictions [7,9]. More detailed lattice study would be desirable to clarify the significance of those observations.

4. Subsequent lattice studies

There are four subsequent lattice studies of pentaquark spectroscopy to be found in the literature [18–21]. Other two preliminary results had been also reported at some conferences [22,23]. I give a short review of those results as follows.

Recently, Kentucky group performed their simulations near the physical pion mass region with overlap fermions [18]. However, they choose *the simple minded operator as the product of nucleon and kaon operators* to explore the pentaquark baryons. Instead, the sequential constrained fitting method is applied in their analysis to disentangle the pentaquark signal from towers of KN scattering state. They also check carefully the volume dependence of the spectral weight. They claim as follows: Only towers of KN scattering state are seen in the negative parity channel. They confirmed that ground state in either parity channels has a characteristic volume dependence on the spectral weight, which should have $1/L^3$ dependence for two particles. They also confirm that the ghost contribution from $KN\eta'$ state appears from pion mass less than around 0.3 GeV in the positive parity channel. Their final conclusion is that there is no sign of pentaquark signal in either parity channels in their

³See a footnote in Ref. [16].

Table 1
Summary of the present status of lattice pentaquark spectroscopy

author(s) and reference	signal	parity of pentaquark	kind of operator
Csikor <i>et al.</i> [16]	Yes	negative	color variant of KN
Sasaki [17]	Yes	negative	diquark-diquark-antiquark
Mathur <i>et al.</i> [18]	No	N/A	simple KN
Chiu-Hsieh [19]	Yes	positive	diquark-diquark-antiquark
Ishii <i>et al.</i> [20]	No	N/A	diquark-diquark-antiquark
Alexandrou <i>et al.</i> [21]	Yes	negative	diquark-diquark-antiquark
MIT group [22]	Yes	negative	diquark-diquark-antiquark
YITP group [23]	Yes	negative	simple KN and color variant of KN

calculation. However, it seems that their results are consistent with the experimental fact that the $\Theta^+(1540)$ state has not yet been found in the KN scattering data as an elastic resonance [24].

Another negative results against the first two studies are reported by TIT group [20]. First, they trace calculations of Ref. [17] on an anisotropic lattice with the $O(a)$ improved Wilson fermion action. They confirm that the lowest energy state appears in the negative parity channel. In their study, a new method is proposed to lift up the S-wave KN threshold by imposing a hybrid boundary condition (HBC) in the spatial direction and it is also applied in their calculation. They found that the plateau of the lowest energy state is shifted in the effective mass plot by changing the spatial boundary condition from PBC to HBC as is expected in the case of the two hadron states. However, their calculations are employed with smeared sources, which are apparently optimized for the ground state of usual hadrons, *i.e.* nucleon and kaon. This optimization may enhance the signal of the KN scattering states rather than that of the pentaquark state. Indeed, the resulting effective mass plots for the pentaquark do not show any effectiveness of their smearing in the earlier Euclidean time region, in contrast to the case of usual baryons. However, of course, their method, *i.e.* the hybrid boundary method, is quite promising to distinguish between the pentaquark state and the KN scattering state.

There are several positive results for the first

two studies. Cyprus group performed careful studies concerning the volume dependence of the spectral weight and then found that their observed state seems to be a single hadron state, *i.e.* the pentaquark state [21]. MIT group formulates eight possible local operators based on the diquark-diquark-antiquark structure to perform the 4×4 correlation matrix analysis in both isosinglet and isotriplet channels as a extension of Sasaki's proposal [22]. Their preliminary results are currently obtained from double exponential fits on data for each single operator. They confirmed results of the first two studies at their exploratory stage. YITP group has started the 2×2 correlation matrix analysis with the simple minded operator as the product of nucleon and kaon operators and its color variant [23]. Their preliminary results support that there is an indication of the presence of the pentaquark state near the lowest KN scattering state in the negative parity channel.

Finally, I comment on Chiu-Hsieh's results. In the first version of Ref. [19], they initially misunderstood the parity assignment. Then, their results are completely opposite to ones of any other studies where the lowest energy state appears in the negative parity channel. In the second version, they corrected this apparent error and reanalyzed their data. However, their final conclusion remains unchanged. They insist that the spin-parity of the $\Theta^+(1540)$ state is most likely $(1/2)^+$. This conclusion seems to attribute to their crude chiral extrapolation.

5. Summary and Outlook

Table 1 represents a summary of the present status for each lattice calculation. The first conclusion of the first two studies as summarized in Sec. 3.2 is confirmed by subsequent lattice studies. The currently important issue is whether or not to establish the presence of the $\Theta^+(1540)$ in the negative parity channel. It is necessary for this to disentangle the pentaquark signal from the KN scattering states completely. The correlation matrix analysis is strongly required to separate the KN scattering state and isolate the pentaquark state. We also should check the volume dependence of the spectral weight. Probably, the hybrid boundary condition is helpful to identify the pentaquark state as a single hadron state. Needless to say, we ought to try non-local types of pentaquark operator in order to verify whether there is no indication of the $\Theta^+(1540)$ in the positive parity channel.

Finally I stress that all present results should be regarded as exploratory. Indeed, much detail studies are in progress in each group. Thus, I had better to conclude that the following questions still remain open:

- Does the spectrum of QCD possess the $\Theta^+(1540)$?
- What is spin and parity of the $\Theta^+(1540)$?
- Are there other pentaquark baryons, *e.g.* the charm (bottom) pentaquark or the spin-orbit partner of the $\Theta^+(1540)$?

There are many exiting issues to be explored.

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